

1 Arithmetic with “*” Operations Oct 2025 (No Calculators)

3 pts 1. If $x * y = 4x^y - 3xy - 2y^x$, find $(3 * 2) * 3$.

Ans. _____

4 pts 2. Find the value in simplest form of

$$\left[\left(\frac{3}{2} \right)^{-3} + \left(\frac{9}{4} \right)^{-1} \right]^{-1} + 2^{-3}$$

Ans. _____

5 pts 3. In the equation $\left(-\frac{2}{3}\right)\left(\frac{3}{x}\right)^{-2} = \left(\frac{y}{2}\right)^{-3}$, when $x = 2$, $y = m$.

If $x \nabla y = x^3y - y^2x^2 + 5xy^3$, find $2 \nabla m$.

Ans. _____

2 Inequalities and Absolute Oct 2025 (No Calculators)

3 pts 1. Find the product of the largest and the smallest integer solutions to

$$|2x - 7| < 10.$$

Ans. _____

4 pts 2. Solve for n :

$$(n - 1)^4 > 16/81$$

Ans. _____

5 pts 3. Solve the following for x : $\frac{5}{2x-1} \geq \frac{x}{x+4}$.

Ans. _____

3 Matrices, Determinants and Systems of Equations Oct 2025 (No Calculators)

3 pts 1. Solve the following system

$$\begin{aligned} 2x - 5y &= -33 \\ 3x + 4y &= 8 \end{aligned}$$

Give answer in ordered pair (x, y) form.

Ans. _____

4 pts 2. If $2x + y = 3$ and $5y + z = 67$ and $7x - 3z = -18$, find the value of $z - y - x$.

Ans. _____

5 pts 3. Solve for x :

$$\begin{vmatrix} x & -2 & 3 \\ 1 & 4 & x \\ -5 & -3 & x \end{vmatrix} = \begin{vmatrix} 13x & 9 \\ -8 & 2 \end{vmatrix}$$

Ans. _____

4 Number Theory Oct 2025 (No Calculators)

3 pts 1. Find the sum of all prime numbers less than 100 that end in 9.

Ans. _____

4 pts 2. Find the remainder in base 10 for

$$(2344_5 + 1432_5) \div 1001_2$$

Ans. _____

5 pts 3. Find the remainder when $87^{39} + 83^{78}$ is divided by 10.

Ans. _____

5 Geometric Similarities Oct 2025 (You may use calculators)

3 pts 1. A triangle with side lengths of 5, 11 and 15 is similar to another triangle whose longest side is 24. Find the sum of the other two sides of the other triangle.

Ans. _____

4 pts 2. A 6-foot man, standing 18 feet from a lamppost, casts a 9-foot shadow. How far must he walk away from the lamppost, towards his shadow, in order to cast an 11-foot shadow?

Ans. _____

5 pts 3. A 10 cm radius metal sphere weighs 10 kg. How much will a 12 cm radius metal sphere weigh, of the same material?

Ans. _____

6 Team Oct 2025 (You may use calculators)

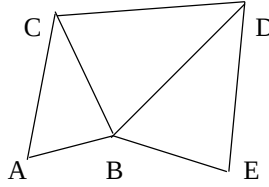
3 pts 1. $a * b = \frac{3a^2 - b^3}{b^2 - a^3}$, find $(-1 * 2) * -3$.

(1) Ans. _____ **3 pts**

3 pts 2. $\triangle ABC \sim \triangle BCD \sim \triangle EBD$

$AB = 18, BC = 24$ and $AC = 30$.

Find the perimeter of $\triangle EBD$.



(2) Ans. _____ **3 pts**

3 pts 3. If h, t and u are unique positive integers, find the least value for n such that

$n = 49h + 7t + u$ and $h + t + u = 7$.

(3) Ans. _____ **3 pts**

4 pts 4. Solve for x :
$$\begin{vmatrix} 3x & 4 & -2x \\ -2 & x & -4 \\ 1 & 2 & -1 \end{vmatrix} = 63$$

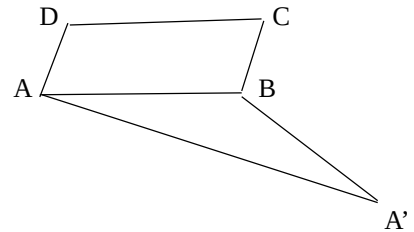
(4) Ans. _____ **4 pts**

4 pts 5. Find all the values of x that will satisfy $|3x + 5| \leq |4x - 8|$.

(5) Ans. _____ **4 pts**

4 pts 6. In parallelogram $ABCD$, $m\angle DAB = 60^\circ$, $AB = 12$.

A' is the reflection of A over line BC . If the area of $\triangle ABA'$ is equal to the area of parallelogram $ABCD$, find AD .



(6) Ans. _____ **4 pts**

5 pts 7. Find the product of x and y , if

$$\begin{vmatrix} x & 2 & 1 \\ -2 & 4 & -1 \\ y & 2 & 4 \end{vmatrix} = \begin{vmatrix} -1 & y \\ 15 & x \end{vmatrix} \text{ and } \begin{vmatrix} -8 & -4 \\ x & y \end{vmatrix} - \frac{5}{2} \begin{vmatrix} y & x \\ 6 & -2 \end{vmatrix} = -72.$$

(7) Ans. _____ **5 pts**

5 pts 8. Find the sum of the integers from 1 to 100 which are not divisible by any of the integers from 2 through 10.

(8) Ans. _____ **5 pts**

5 pts 9. For what values of x will $\frac{3}{2x-5} \leq \left| \frac{5}{3x+7} \right|$?

(9) Ans. _____ **5pts**

Solutions - Arithmetic with “*” Operations

1. $3 * 2 = 4(3)^2 - 3(3)(2) - 2(2)^3 = 36 - 18 - 16 = 2$. $2 * 3 = 4(2)^3 - 3(2)(3) - 2(3)^2 = 32 - 18 - 18 = -4$. **Ans. - 4**

2. $\left[\left(\frac{3}{2}\right)^{-3} + \left(\frac{9}{4}\right)^{-1}\right]^{-1} + 2^{-3} \rightarrow \left[\frac{8}{27} + \frac{4}{9}\right]^{-1} + \frac{1}{8} = \left(\frac{20}{27}\right)^{-1} + \frac{1}{8} = \frac{27}{20} + \frac{1}{8} = \frac{5+54}{40} = \frac{59}{40}$. **Ans. 59/40**

3. $\left(-\frac{2}{3}\right)\left(\frac{3}{x}\right)^{-2} = \left(\frac{y}{2}\right)^{-3}$, when $x = 2$: $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right)^{-2} = \left(\frac{y}{2}\right)^{-3} \rightarrow \left(-\frac{2}{3}\right)\left(\frac{4}{9}\right) = \frac{8}{y^3} \rightarrow y = -3$.

So $2 \nabla -3 = 2^3(-3) - (-3)^2(2)^2 + 5(2)(-3)^3 \rightarrow -24 - 36 - 270 = -330$. **Ans. - 330**

Inequalities and Absolute Values

1. $-10 < 2x - 7 < 10 \rightarrow -3 < 2x < 17 \rightarrow -3/2 < x < 17/2$. Largest = 8, smallest -1. **Ans. -8**

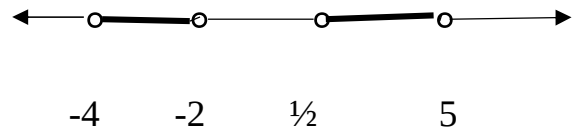
2. $(n-1)^4 > 16/81 \rightarrow (n-1) > \pm 2/3 \rightarrow$ Critical points are at $n = 1 \pm 2/3$ or $n = 1/3$ or $5/3$.

Plugging in to intervals: $0 \rightarrow 1 > 16/81$, yes; $1 \rightarrow 0 > 16/81$, no; $2 \rightarrow 1 > 16/81$, yes.

Ans. $n < 1/3$ or $n > 5/3$

3. Critical points are at $1/2$ and -4 and where $5x + 20 = 2x^2 - x \rightarrow 0 = 2x^2 - 6x - 2 \rightarrow$

$x^2 - x - 10 = 0$, $(x-5)(x+2) = 0$, $x = 5$ or -2 .



Checking interval points using $\frac{5}{2x-1} \geq \left|\frac{x}{x+4}\right|$: $-5 \rightarrow -5/11 \geq 5$, no; $-3 \rightarrow -5/7 \geq -3$, yes;

$0 \rightarrow -5 \geq 0$, no; $1 \rightarrow 5 \geq 1/5$, yes; $6 \rightarrow \frac{5}{11} \geq \frac{3}{5}$, $25 \geq 33$, no.

Ans. $-4 < x \leq -2$ or $1/2 < x \leq 5$

Matrices, Determinants and Systems of Equations

1. $2x - 5y = -33 \rightarrow -6x + 15y = 99$ Adding we get $23y = 115 \rightarrow y = 5$. Subbing: $3x + 20 = 8$

$3x + 4y = 8 \rightarrow 6x + 8y = 16$ $3x = -12$, so $x = -4$ **Ans. (-4, 5)**

2. $2x + y = 3 \rightarrow y = 3 - 2x$. $5y + z = 67 \rightarrow 5(3 - 2x) + z = 67 \rightarrow 15 - 10x + z = 67$

$-10x + z = 52 \rightarrow -30x + 3z = 156$ Adding: $-23x = 138$, so $x = -6$. $2(-6) + y = 3$, so $y = 15$.

$7x - 3z = -18 \rightarrow 7x - 3z = -18$ $-10(-6) + z = 52$, so $z = -8$.

$z - y - x = -8 - (15) - (-6) = -17$.

Ans. - 17

$$3. \begin{vmatrix} x & -2 & 3 \\ 1 & 4 & x \\ -5 & -3 & x \end{vmatrix} = \begin{vmatrix} 13x & 9 \\ -8 & 2 \end{vmatrix} \Rightarrow 4x^2 + 10x - 9 + 60 + 3x^2 + 2x = 26x + 72 \Rightarrow$$

$$7x^2 + 12x + 51 = 26x + 72 \Rightarrow 7x^2 - 14x - 21 = 0 \Rightarrow x^2 - 2x - 3 = (x - 3)(x + 1) = 0, \text{ thus}$$

$$x = 3 \text{ or } x = -1.$$

Ans. x = 3 or -1

Number Theory

1. There are 5 such primes: 19, 29, 59, 79 and 89. Their sum is $(20 + 30 + 60 + 80 + 90) - 5$ equals $280 - 5 = 275$.

Ans. 275

2. Adding: $2344_5 + 1432_5 = 4331_5 \Rightarrow 4(125) + 3(25) + 3(5) + 1 = 500 + 75 + 15 + 1 = 591$.

$1001_2 = 8 + 1 = 9$. Dividing 591 by 9 gives a remainder of $5 + 1 = 6$.

Ans. 6

3. 87^{39} , pattern for powers of 7 are 7, 9, 3, 1. Dividing 39 by 4 gives remainder of 3, that digit is 3. 83^{78} , pattern for powers of 3 are 3, 9, 7, 1. Dividing 78 by 4 gives a remainder of 2, that digit is 9. $3 + 9 = 12$. Dividing by 10 gives a remainder of 2.

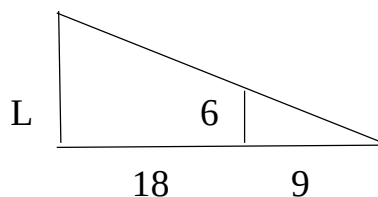
Ans. 2

Geometric Similarities

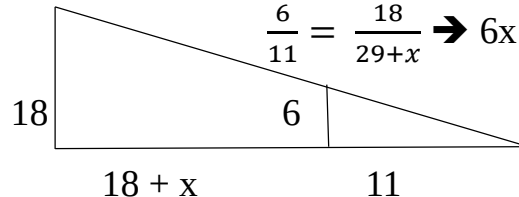
1. Scale factor: $\frac{\text{other triangle}}{\text{given triangle}} = \frac{24}{15} = \frac{8}{5}$. $\frac{8}{5}(5 + 11) = 128/5 = 25.6$.

Ans. 25.6

2. $\frac{L}{27} = \frac{6}{9}$, $L = 18$



$\frac{6}{11} = \frac{18}{29+x} \Rightarrow 6x + 174 = 198$



$$6x = 24$$

$$x = 4$$

Ans. 4 ft

3. $\frac{x \text{ kg}}{10 \text{ kg}} = \left(\frac{12}{10}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$, thus $x = \frac{2160}{125} = 17.28 \text{ kg}$.

Ans. 17.28 kg

Team

1. $a * b = \frac{3a^2 - b^3}{b^2 - a^3}$, thus $(-1 * 2) = \frac{3(-1)^2 - (2)^3}{(2)^2 - (-1)^3} = \frac{-5}{5}$. $-1 * -3 = \frac{3(-1)^2 - (-3)^3}{(-3)^2 - (-1)^3} = \frac{30}{10} = 3$.

Ans. 3

2. $\frac{18}{30} = \frac{24}{BD} \Rightarrow 18BD = 24 \cdot 30 \Rightarrow BD = \frac{24 \cdot 30}{18} = \frac{4 \cdot 30}{3} = 40$. Since $\triangle ABC$ is a basic 3-4-5 Δ , then the sides of $\triangle EBD$ must be 30-40-50. The perimeter is 120.

Ans. 120

3. Since h, t and u are unique positive integers and $h + t + u = 7$, then their values could only be 4, 2 and 1. For the least value of n, h would have to be 1, t would have to be 2 and u would have to be 4. Thus $49(1) + 7(2) + 4 = 67$.

Ans. 67

$$4. \begin{vmatrix} 3x & 4 & -2x \\ -2 & x & -4 \\ 1 & 2 & -1 \end{vmatrix} = 63 \rightarrow -3x^2 - 16 + 8x + 2x^2 + 24x - 8 = 63 \rightarrow -x^2 + 32x - 24 = 63 \rightarrow$$

$$x^2 - 32x + 87 = 0 \rightarrow (x - 29)(x - 3) = 0, \text{ so } x = 29 \text{ or } 3.$$

Ans. 29 or 3

5. Crit. Pts: $3x + 5 = 4x - 8$, $x = 13$; or $3x + 5 = 8 - 4x$, $7x = 3$, $x = 3/7$. Plugging interval points into $|3x + 5| \leq |4x - 8|$: $0 \rightarrow 5 \leq 8$, yes; $1 \rightarrow 8 \leq 4$, no; $16 \rightarrow 53 \leq 56$, yes.

Ans. $x \leq 3/7$ or $x \geq 13$

6. Extend $\overline{BC}$ through B to intersect $\overline{AA'}$ at E. Since $\overline{AD} \parallel \overline{CE}$, then $\angle DAB \cong \angle ABE$. A line joining a point and its reflection is perpendicular to the line of reflection, so $\angle BEA$ is a right angle. Since $\angle BEA$ is a right angle, then ADBE must be a rectangle. $\triangle ABE$ is a 30-60-90 $\triangle$, if $AB = 12$, then $BE = 6$ and since $\triangle ADB \cong \triangle BEA$, then $AD = 6$.

Ans. 6

$$7. \begin{vmatrix} x & 2 & 1 \\ -2 & 4 & -1 \\ y & 2 & 4 \end{vmatrix} = \begin{vmatrix} -1 & y \\ 15 & x \end{vmatrix} \rightarrow 16x - 2y - 4 - 4y + 2x + 16 = -x - 15y \rightarrow 18x - 6y + 12 = -x$$

$$-15y \rightarrow (1) 19x + 9y = -12. \begin{vmatrix} -8 & -4 \\ x & y \end{vmatrix} - \frac{5}{2} \begin{vmatrix} y & x \\ 6 & -2 \end{vmatrix} = -72 \rightarrow -8y + 4x - \frac{5}{2}(-2y - 6x) = -72$$

$$\rightarrow -8y + 4x + 5y + 15x = -72 \rightarrow (2) 19x - 3y = -72. (1) - (2): 12y = 60, \text{ so } y = 5. \text{ In (1):}$$

$$19x + 9(5) = -12, 19x = -57, \text{ so } x = -3. xy = -15.$$

Ans. -15

8. The numbers are (1) + (11, 13, 17, 19) + (23, 29, 31, 37) + (41, 43, 47, 53, 59) +

$$(61, 67, 71, 73, 79) + (83, 89, 97) \rightarrow (1) + (30 + 30) + (60 + 60) + (100 + 100 + 43) + (140 + 140 + 71) + 180 + 89 \rightarrow 1 + 60 + 120 + 200 + 43 + 280 + 71 + 180 + 89 \rightarrow 380 + 44 + 460 + 160 \rightarrow 380 + 44 + 620 = 1044.$$

Ans. 1044

9. Critical points for $\frac{3}{2x-5} \leq \left| \frac{5}{3x+7} \right|$ are at $x = 2\frac{1}{2}, -2\frac{1}{3}$ and where (1) $9x + 21 = 10x - 25$ or (2) $9x + 21 = 25 - 10x$. In (1): $x = 46$, In (2): $19x = 4$, or $x = \frac{4}{19}$. Plugging in interval points:

$$-3 \rightarrow -\frac{3}{11} \leq \frac{5}{2}, \text{ yes; } 0 \rightarrow -\frac{3}{5} \leq \frac{5}{7}, \text{ yes; } 1 \rightarrow -1 \leq \frac{1}{2}, \text{ yes; } 10 \rightarrow \frac{1}{5} \leq \frac{5}{37}, 37 \leq 25, \text{ no; } 50 \rightarrow \frac{3}{95} \leq \frac{5}{157}, 471 \leq 475, \text{ yes. Plugging in } 46: \frac{3}{92-5} = \frac{5}{138+7} \rightarrow \frac{3}{87} = \frac{5}{145} \rightarrow \frac{1}{29} = \frac{1}{29} \text{ and } \frac{4}{19}: \frac{3}{\frac{8}{19}-5} = \frac{5}{\frac{12}{19}+7}, \text{ no.}$$

Even though $\frac{4}{19}$ will not make an equality, it will satisfy the inequality.



$$-2\frac{1}{3} \quad \frac{4}{19} \quad 2\frac{1}{2} \quad 46$$

Ans. $x < -2\frac{1}{3}$ or $x \geq 46$, but $x \neq -2\frac{1}{3}$

Arithmetic with “*” Operations

Team

1. -4

1. 3

2. $59/40$

2. 120

3. -330

3. 67

Inequalities and Absolute Values

4. 29 or 3

1. -8

5. $x \leq 3/7$ or $x \geq 13$

2. $n < 1/3$ or $n > 5/3$

6. 6

3. $-4 < x \leq -2$ or $1/2 < x \leq 5$

7. -15

Matrices, Determinants and Systems of Equations

8. 1044

1. $(-4, 5)$

9. $x < 2\frac{1}{2}$ or $x \geq 46$, but $x \neq -2\frac{1}{3}$

2. -17

3. 3 or -1

Number Theory

1. 275

2. 6

3. 2

Geometric Similarities

1. 25.6 ($25\frac{3}{5}$) ($128/5$)

2. 4 ft (4)

3. 17.28 kg ($17\frac{7}{25}$ kg) ($432/25$ kg) (17.28) ($17\frac{7}{25}$) ($432/25$)

NOTE: Interval Notation for the following answers is acceptable

Round 2 #2: $(-\infty, \frac{1}{3}) \cup (\frac{5}{3}, \infty)$

Round 2 #3: $(-4, -2] \cup (\frac{1}{2}, 5]$

Team #5: $(-\infty, \frac{3}{7}] \cup [13, \infty)$

Team #9: $(-\infty, -\frac{7}{3}) \cup (-\frac{7}{3}, \frac{5}{2}) \cup [46, \infty)$

Additional answers accepted on appeal:

Round 5 #1: **25.6000**

Round 5 #2: **22**

Round 5 #3: **17.2800**