

1 Arithmetic with Ratio and Proportion Nov 2025 (No Calculators)

3 pts 1. A certain school has a total of 405 teachers and students in a ratio of 1:3:5, teachers to boy students to girl students, respectively. How many more girl students than boy students are there?

Ans. _____

4 pts 2. Two farmers, Angela and Bob, own 1000 acres of land in the ratio of 3 to 1, respectively. How many acres must Angela sell to Bob so that the resulting ratio will be 5 to 3, with Angela still having more?

Ans. _____

5 pts 3. n varies jointly with x and the square of y , and inversely with the cube of z . When $x = 3$ and $y = 4$ and $z = 6$, $n = 10/3$. Find the reciprocal of x , when $n = 10/9$, $y = 15$ and $z = 3$.

Ans. _____

2 Series and Sequences Nov 2025 (No Calculators)

3 pts 1. In the arithmetic sequence below, which begins with 3, which term is 267?

3, 27, 51, ...

Ans. _____

4 pts 2. Find the 6th term in the geometric sequence where 8 is the first term and $81/2$ is the 5th term.

Ans. _____

5 pts 3. Find the possible value(s) of the infinite geometric series, where $a_3 = 36$ and $a_5 = 20\frac{1}{4}$.

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2025 (No Calculators)

3 pts 1. A basketball team has 10 players. However, only 5 can play at a time. Assuming any player can play all positions (so position assignments are irrelevant), how many teams of 5 players are possible?

Ans. _____

4 pts 2. Find the sum of the coefficients of the terms containing x^3 and the x^4 in the expansion of $(3x - 2y)^5$.

Ans. _____

5 pts 3. In how many distinguishable ways can the letters d, i, v, i, d, e, d be written?

Ans. _____

4 Polynomials Nov 2025 (No Calculators)

3 pts 1. Find all real zeros of $f(x)$, if $f(x) = x^3 + 5x^2 - 6x$.

Ans. _____

4 pts 2. $h(x) = x^3 - 5x^2 + kx + 8$. Find the value of k so that $(x + 1)$ is a factor of $h(x)$.

Ans. _____

5 pts 3. If $f(x) = 8x^5 + 6x^4 - 37x^3 - 36x^2 + 29x + 30$ and $g(x) = 8x^2 + 22x + 15$, find all zeros of $r(x)$, where $\frac{f(x)}{g(x)} = r(x)$.

Ans. _____

5 Areas and Volumes Nov 2025 (You may use calculators)

3 pts 1. The radius of a right cylinder is 2 inches and its height is 5 inches. A second cylinder has a radius of 5 inches and a height of 2 inches. What is the ratio of the volume of the smaller cylinder to that of the larger cylinder?

Ans. _____

4 pts 2. A large cylindrically shaped water container has a 9-inch diameter and a height of 24 inches. Drinking cups are in the shape of a cone whose radius of base is 1.5 inches and are 3 inches high. The container is 75% full. How many full cups of drinking water will it take to consume $\frac{5}{6}$ of the water in the container?

Ans. _____

5 pts 3. A target is made up of 7 squares, all centered around the same point in the same plane with all sides either vertical or horizontal. The regions between the squares alternate white and blue, with the smallest square having a white interior. The sides of the regions are the same distance apart. The smallest square has an area of 4.41 dm^2 and the largest square has an area of 151.29 dm^2 . Find the area of the third white region from the center of the target, where the smallest region is the first.

Ans. _____

6 Team Nov 2025 (You may use calculators)

3 pts 1. A square and a circle have equal areas. Compute the perimeter of the square, if the circle has a circumference of 100π .

(1) Ans. _____ 3 pts

3 pts 2. Find the coefficient of the x^2 term in the polynomial that results from expanding $(x - 2)(x + 3)(x + 2)(x - 3)$ completely.

(2) Ans. _____ 3 pts

3 pts 3. If the 13th term of an arithmetic sequence is 208 and the 17th term is 292, what is the 35th term?

(3) Ans. _____ 3 pts

4 pts 4. In the binomial expansion of $(x + 1)^n$, what is the positive difference between the sum of all the coefficients of the expansion when $n = 12$ and when $n = 8$.

(4) Ans. _____ 4 pts

4 pts 5. The sum of 25 terms of an arithmetic sequence is 2025. The first term is 20. Find the sum of the middle term and the 25th term.

(5) Ans. _____ 4 pts

4 pts 6. Rewrite the base 2 number 11,111,101,001 in the smallest base that requires exactly 6 digits.

(6) Ans. _____ 4 pts

5 pts 7. A ball is dropped from a height of 24 ft and rebounds 75% of its height. What total distance has the ball traveled when it hits the ground for the fourth time?

(7) Ans. _____ 5 pts

5 pts 8. A right circular cone, height 12 in., and radius 5 in., is cut into 3 pieces by 2 planes parallel to the base. The plane closest to the base is a inches from the base and the other plane is b inches from the base. If the three pieces (1 cone and 2 frustums) each have the same volume, find the value of $b - a$.

(8) Ans. _____ 5pts

5 pts 9. Let $a_1, a_2, a_3, a_4, \dots$ be a geometric sequence. Find $a_1 + a_2$, if given $a_4 - a_3 = 216$ and $a_7 - a_6 = 5,832$.

(9) Ans. _____ 5 pts

Solutions – Arithmetic with Ratio and Proportion

1. Ratio total is 9. $9x = 405$, each part is 45. $5(45) - 3(45) = 2(45) = 90$. **Ans. 90**

2. $4x = 1000$, Bob has 250 acres, Angela has 750. $\frac{750-x}{250+x} = \frac{5}{3} \rightarrow 2250 - 3x = 1250 + 5x \rightarrow$

$1000 = 8x$, so $x = 125$. **Ans.**

125

3. $\frac{nz^3}{xy^2} \rightarrow \frac{\frac{10}{3}(6^3)}{3 \cdot 4^2} = \frac{\frac{10}{9}(3^3)}{x \cdot 15^2} \rightarrow \frac{10 \cdot 2^3 \cdot 3^3}{3^2 \cdot 2^4} = \frac{10 \cdot 3^3}{3^2 \cdot x \cdot 3^2 \cdot 5^2} \rightarrow \frac{2^2 \cdot 3^2 \cdot 5^2}{10 \cdot 3^3} \cdot \frac{10 \cdot 2^3 \cdot 3^3}{3^2 \cdot 2^4} = \frac{1}{x} \rightarrow \frac{225}{2} = \frac{1}{x}$. **Ans. $\frac{225}{2}$**

Series and Sequences

1. $267 = 3 + (n-1)24 \rightarrow 267 = 3 + 24n - 24 \rightarrow 288 = 24n$, $n = 12$. **Ans. 12**

2. $\frac{81}{2} = 8r^{5-1} \rightarrow \frac{81}{2} = 8r^4 \rightarrow \frac{81}{16} = r^4$, so $r = \pm \frac{3}{2}$. $a_5(r) = r_6 \rightarrow (\pm \frac{3}{2}) \frac{81}{2} = \pm \frac{243}{4}$. **Ans. $\pm \frac{243}{4}$**

3. $36r^2 = \frac{81}{4} \rightarrow r^2 = \frac{81}{144}$, $r = \pm \frac{3}{4}$. $a_1 = a_3(\pm \frac{4}{3})^2$, $a_1 = 36(\frac{16}{9}) = 64$. $S = \frac{64}{1 \pm \frac{3}{4}} = \frac{64}{\frac{1}{4}}$ or $\frac{64}{\frac{7}{4}} = 256$ or $\frac{256}{7}$. **Ans. 256 or $\frac{256}{7}$**

Counting Principles and Binomial Theorem

1. ${}_{10}C_5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 5 \cdot 4 \cdot 3 \cdot 2!} = 3(2)(7)(6) = 36(7) = 252$. **Ans. 252**

2. $(3x - 2y)^5$ The x^3 term: $\binom{5}{2} (3x)^3 (-2y)^2 = 10(27x^3)(4y^2)$, coefficient = 1080. The x^4 term: $\binom{5}{4} (3x)^4 (-2y)^1 = 5(81x^4)(-2y)$, coefficient = -810. $1080 - 810 = 270$. **Ans. 270**

3. divided has 7 letters, 3 of which are d's and 2 are i's. $\frac{7!}{3!2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2} = 7(6)(5)(2)$ **Ans. 420**

Polynomials

1. $f(x) = x^3 + 5x^2 - 6x = x(x^2 + 5x - 6) = x(x+6)(x-1)$. Zeros are 0, -6, 1. **Ans. 0, 1, -6**

2. $-1 \) \ 1 \ -5 \ k \ \quad 8$ Since $x+1$ is a factor then $2-k$ must equal 0. So $k=2$.

$$\begin{array}{r} 0 \ -1 \ \quad 6 \ \quad (-6-k) \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ -6 \ (k+6) \ (2-k) \\ \hline \end{array}$$

Ans. 2

3. Dividing you will get $x^3 - 2x^2 - x + 2$, which factors to $x^2(x-2) - 1(x-2)$ and to $(x^2-1)(x-2)$ and lastly to $(x+1)(x-1)(x-2)$. So the zeros are -1, 1, 2 **Ans. -1, 1, 2**

Areas and Volumes

1. $\frac{\text{Volume of Smaller}}{\text{Volume of Larger}} = \frac{\pi 2^2 \cdot 5}{\pi 5^2 \cdot 2} = \frac{2}{5}$. **Ans. $\frac{2}{5}$**
2. The volume of water: $\pi(4.5)^2(24)(.75) = 364.5\pi \text{ in}^3$. Volume of cone: $\frac{1}{3}(1.5)^2(3) = 2.25\pi$
 # of cups: $\frac{(\frac{5}{6})(364.5)\pi}{2.25\pi} = 135$ cups. **Ans. 135 cups**
3. The side of the smallest square is $\sqrt{4.41} = 2.1$ dm. The largest square's side is $\sqrt{151.29} = 12.3$ dm. The distance between each region is $(12.3 - 2.1)/6 = 1.7$ dm. The area of the 3rd white region from the center is the area of the 5th square minus the area of the 4th square. The 5th square has a side of $12.3 - 3.4 = 8.9$. The side of the 4th square is $8.9 - 1.7 = 7.2$. Thus the area of the region is $8.9^2 - 7.2^2 = 27.37\text{dm}^2$. **Ans. 27.37**

Team

1. Circumference of circle is 100π , radius = 50, area = 100π . Area of square is x^2 , $x^2 = 100\pi$.
 The side of the square is $50\sqrt{\pi}$. The perimeter is $200\sqrt{\pi}$. **Ans. $200\sqrt{\pi}$**
2. Using synthetic multiplication for the first two factors and the last two factors are as follows:
- | | |
|--------------|--|
| 1 1 -6 | Since this is a fourth degree polynomial, the first |
| 1 -1 -6 | sum is the coefficient of the x^4 term, the second |
| -6 -6 36 | is for the x^3 term and the third is for the x^2 |
| -1 -1 6 | term. The sum is -13. |
| 1 1 -6 | |
- Ans. - 13**
3. (1): $208 = a + 12d$ and (2): $292 = a + 16d$. (2) - (1) $\Rightarrow 84 = 4d$, $21 = d$. In (1):
 $208 = a + 12(21) \Rightarrow 208 = a + 252$, $a = -44$. The 35th term: $L = -44 + 34(21) = -44 + 714 = 670$.
Ans. 670
4. The sum of the coefficients of $(x + 1)^n$ for any value of n is 2^n . $2^{12} - 2^8 = 4096 - 256 = 3840$.
Ans. 3840
5. $2025 = \frac{25}{2}(20 + 20 + 24d) \Rightarrow 2025 = 25(20 + 12d) \Rightarrow 2025 = 500 + 300d \Rightarrow 1525 = 300d$,
 so $d = 5\frac{1}{12}$. $a_{13} = 20 + 12(5\frac{1}{12}) = 20 + 61 = 81$. $A_{25} = 20 + 24(5\frac{1}{12}) = 20 + 122 = 142$. The sum
 is $81 + 142 = 223$. **Ans. 223**

6. The base two number converts to 2025 in base 10. This converts to 31100 in base 5, not 6 digits. It converts to 133, 221 in base 4, 6 digits. It converts to 2210000 in base 3, 7 digits.

Ans. 133,221₄

7. The ball hits the ground from 24 ft. It hits the ground after rebounding 18 ft. (36 ft.) It hits the ground again after rebounding 13.5 ft (27 ft). It hits the ground again after rebounding $10\frac{1}{8}$ ft. (20.25 ft.). Total

Ans. 107.25 ft.

8. The volume of the large cone is $\frac{1}{3}(25\pi)12 = 100\pi$. The Volume of the small cone at the top is $100\pi/3$. The height of the small cone: $\frac{1}{3}h(\frac{5}{12}h)^2\pi = \frac{100\pi}{3} \Rightarrow h^3 = \frac{100}{3}(\frac{144 \cdot 3}{25}) = 4(144) = 576$, $h = \sqrt[3]{576} = 8.32034$. So $b = 12 - 8.32034 = 3.67966$. The smallest cone and the smaller frustum form another cone. We perform a calculation similar to the one just completed to find the height of this cone: $\frac{1}{3} \cdot \frac{25}{144}h^3\pi = \frac{200\pi}{3} \Rightarrow h^3 = \frac{200}{3} \cdot \frac{144 \cdot 3}{25} = 8(144) = 1152$, $h = 10.48297$. So $a = 12 - 10.48297 = 1.51703$. $b - a = 3.67966 - 1.51703 = 2.16263$.

Ans. 2.1626

9. The 7 terms in the geometric sequence are $a, ar, ar^2, ar^3, ar^4, ar^5, ar^6$. Thus

$$a_4 - a_3 = ar^3 - ar^2 = 216 \Rightarrow ar^2(r - 1) = 216 \Rightarrow r - 1 = \frac{216}{ar^2}. \quad a_7 - a_6 = ar^6 - ar^5 = 5832 \Rightarrow$$

$$ar^5(r - 1) = 5832 \Rightarrow r - 1 = \frac{5832}{ar^5}. \quad \frac{5832}{ar^5} = \frac{216}{ar^2} \Rightarrow 5832ar^2 = 216ar^5 \Rightarrow 27 = r^3, \text{ so } r = 3.$$

$$a \cdot 3^2(3-1) = 216 \Rightarrow 18a = 216, a = 12. \quad ar = 3(12) = 36. \quad 12 + 36 = 48.$$

Ans. 48

Arithmetic with Ratio and Proportion

1. 90
2. 125 (125 acres)
3. 225/2 (112.5)

Series and Sequences

1. 12
2. $\pm \frac{243}{4}$
3. 256 or 256/7

Counting Principles and Binomial Theorem

1. 252
2. 270
3. 420

Polynomials

1. -6, 0, 1
2. 2
3. -1, 1, 2

Areas and Volumes

1. 2 to 5 (2/5) (2:5)
2. 135 (135 cups)
3. 27.37 (27.37 dm²)

Team

1. $200\sqrt{\pi}$ (354.4908)
2. -13
3. 670
4. 3840
5. 223
6. 133,221₄
7. 107.25
8. 2.1626
9. 48

Additional answers accepted on appeal:

Round 2 #2: Both the individual answers $\frac{243}{4}$ and $-\frac{243}{4}$ were accepted, as it was felt the wording of the problem might have implied a single answer.