

1 Probability Dec 2025 (No Calculators)

3 pt 1. Two digits are selected at random, without replacement, from the digits 1, 2, 4 and 8. What is the probability that the sum of the digits is a multiple of 3?

Ans. _____

4 pts 2. The probability that a two-digit prime will have each digit prime is written as p/q , where p and q are relatively prime. Find the value of $q - p$.

Ans. _____

5 pts 3. Allie and Bobbie have two fair tetrahedral dice. One die has faces of 2, 3, 5 and 7. The other has faces of 3, 5, 7 and 11. Players take turns tossing both dice, until one or the other wins, with a win defined as follows: Allie wins if the sum on the faces is prime, and Bobbie wins if the sum of the faces is a perfect square or a perfect cube. If Allie tosses first, find the probability that Allie wins on her second toss?

Ans. _____

2 Exponents and Radicals Dec 2025 (No Calculators)

3 pts 1. Solve for x over the real numbers: $5^{x-1} = 125^{2x+3}$.

Ans. _____

4 pts 2. Solve for x in the equation $\sqrt{x+8} + \sqrt{x-32} = 20$.

Ans. _____

5 pts 3. Solve the following over the set of real numbers:

$$4x^{\frac{4}{5}} - 5x^{\frac{2}{5}} + 1 = 0$$

Ans. _____

3 Lines, Angles and Polygons Dec 2025 (No Calculators)

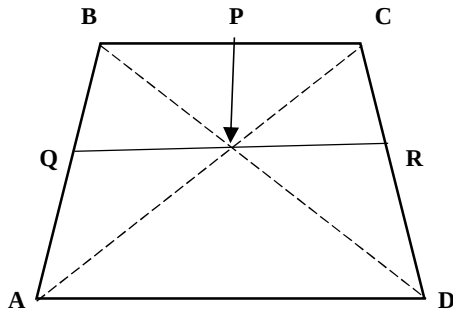
3 pts 1. What is the sum of the measures of the interior angles of a convex pentagon?

Ans. _____

4 pts 2. Each angle of a regular n -gon has a measure of $178\frac{2}{7}$ degrees. A side is $2\frac{1}{3}$ cm long. Find the perimeter of the n -gon.

Ans. _____

5 pts 3. The bases of isosceles trapezoid ABCD are 6 and 10. Find the length of segment QR that contains the intersection of the diagonals P, is parallel to the bases, and meets side AB at Q and side CD at R.



Ans. _____

4 Complex Numbers Dec 2025 (No Calculators)

3 pts 1. Given that $i = \sqrt{-1}$, solve the following equation for x : $(x + i)(x - i) = 10$.

Ans. _____

4 pts 2. Express the following in simplest form: $\frac{5+2i}{2-i} - \frac{1-i}{3+4i}$

Ans. _____

5 pts 3. Find the non-real solutions of the following equation

$$3x^3 + 11x^2 + 11x + 15 = 0.$$

Ans. _____

5 Arithmetic with Percent Dec 2025 (You may use calculators)

3 pts 1. The number of pennies in my 10,000 penny coin collection changes weekly. After week 1, the number of pennies increased by 10%. After week 2, it again increased by 10%. For each of the next two weeks, the number of pennies decreased by 10%. How many pennies are in the collection at the end of the four-week period?

Ans. _____

4 pts 2. What percentage of all three-digit positive integers are palindromic? (Palindromic numbers are numbers that read the same forward as backwards)

Ans. _____

5 pts 3. How old would a person have been in the year 2000 on August 4th, if on August 4th in the year n^2 the person celebrated his or her n th birthday? The person was born after 1900.

Ans. _____

6 Team Dec 2025 (You may use calculators)

3 pts 1. A fair 50-sided die, numbered 1 to 50, is rolled once. Compute the probability of rolling a prime number. That is, the top face is a prime number.

(1) Ans. _____ 3 pts

3pts 2. If $(2 + i)(3 - i)(5 + 2i) = A + Bi$, find the value of $A + B$. **(2) Ans. _____ 3 pts**

3 pts 3. Two fair, regular 6-sided dice are rolled. What is the probability that the product of the top faces on the two dice is odd?

(3) Ans. _____ 3 pts

4 pts 4. Three fair, regular 6-sided dice are tossed. What is the probability that the sum of the dice is 6? Give your answer as a fraction.

(4) Ans. _____ 4 pts

4 pts 5. A sequence of isosceles triangles have bases on the positive x -axis and vertices on the line $y = 2x$. The base of the first triangle, T_1 , has endpoints $(0, 0)$ and $(4, 0)$. The second triangle, T_2 , has endpoints $(4, 0)$ and $(8, 0)$, etc. If the endpoints of the base of T_n are $(4n, 0)$ and $(4n + 4, 0)$, what is the area of triangle T_n ? Give your answer in simplest form with all possible calculations completed.

5 Ans. _____ 4 pts

4 pts 6. All possible 3-4-5 triangles are placed on the xy -plane, so that one vertex of each triangle is at the origin and one side lies on the positive y -axis. All the vertices that result are used for a probability event. Although several vertices from different triangles may coincide, such as at the origin, such a vertex is only counted once. If a vertex is chosen at random, what is the probability that it lies outside of the circle $x^2 + y^2 = 16$?

(6) Ans. _____ 4 pts

5 pts 7. Regular polygon ABCDEFGHIJKL has a side length of 16. Compute the area of quadrilateral ABCD.

(7) Ans. _____ 5 pts

5 pts 8. Solve for x , if $(x - 1 - 2i)^4 = 225 - 16(x - 1 - 2i)^2$.

(8) Ans. _____ 5 pts

5 pts 9. $\sqrt{12 - 4\sqrt{5}}$ may be written as $\sqrt{A} - \sqrt{B}$. Find the value of $A - B$

(9) Ans. _____ 5 pts

Solutions – Probability

1. Possible pairs in parentheses are (1,2), (1,4), (1,8), (2,1), (2,4), (2,8), (4,1), (4,2), (4,8), (8,1), (8,2), (8,4). Of these pairs that are not a multiple of 3 are (1,4), (2,8), (4,1) and (8,2). So 8 have a multiple of 3. $8/12 = 2/3$. **Ans. 2/3**

2. There are 21 two-digit primes. The entire list has only four primes which have both digits prime, 23, 37, 53, and 73. Probability = $4/21$. $B - A = 21 - 4 = 17$. **Ans. 17**

3. Possible outcomes are (2,3), (2,5), (2,7), (2,11), (3,3), (3,5), (3,7), (3,11), (5,3), (5,5), (5,7), (5,11), (7,3), (7,5), (7,7), (7,11). There are 16 possible outcomes. 3 are prime sums. The number of perfect squares is 2, the number of perfect cubes is 2. So if Allie does not win on the first try $13/16$. Bobbie does not win on his first try $12/16$ or $3/4$. Allie wins on the second try $3/16$. So final: $\frac{13}{16} \cdot \frac{3}{4} \cdot \frac{3}{16} = 117/1024$. **Ans. 117/1024**

Exponents and Radicals

1. $5^{x-1} = 125^{2x+3} = 5^{6x+9}$. So $x - 1 = 6x + 9 \rightarrow -10 = 5x$, $x = -2$. **Ans. -2**

2. $\sqrt{x+8} + \sqrt{x-32} = 20 \rightarrow \sqrt{x+8} = 20 - \sqrt{x-32}$. Squaring both sides:
 $x + 8 = 400 - 40\sqrt{x-32} + x - 32 \rightarrow 40 = 400 - 40\sqrt{x-32} \rightarrow 1 = 10 - \sqrt{x-32} \rightarrow$
 $\sqrt{x-32} = 9 \rightarrow x - 32 = 81$, so $x = 113$. **Ans. 113**

3. $4x^{\frac{4}{5}} - 5x^{\frac{2}{5}} + 1 = 0$. Let $n = x^{\frac{2}{5}}$, Then $4n^2 - 5n + 1 = 0 \rightarrow (4n - 1)(n - 1) = 0$, $n = \frac{1}{4}$ or 1 .
 $x^{\frac{2}{5}} = \frac{1}{4} \rightarrow (x^2)^{1/5} = 2^{-2} \rightarrow x^2 = 2^{-10} \rightarrow x = \pm 2^{-5}$, so $x = \pm \frac{1}{32}$ or $x = \pm 1$. **Ans. $\pm \frac{1}{32}$ or ± 1**

Lines, Angles and Polygons

1. $360/5 = 72$, the measure of each exterior. $180 - 72 = 108$, measure of each interior angle.
 $5(108) = 540$, sum of all interior angles. **Ans. 540**

2. Let $n = \#$ sides. $360/1\frac{5}{7} = n \rightarrow 360(\frac{7}{12}) = 30(7) = 210$. $210(\frac{7}{3}) = 70(7) = 490$. **Ans. 490**

3. Since $\triangle PBC \sim \triangle PDA$ with a linear ratio of $6/10$ or $3/5$. Let $PB = 3y$ and $PD = 5y$. Therefore $BD = 8y$. Let $x = PR$. Notice that $\triangle DPR \sim \triangle DBC$, so $x/6 = DP/DB = 5y/8y = 5/8$. Thus $x = 6(\frac{5}{8}) = \frac{15}{4}$. $QR = 2PR = 15/2 = 7.5$. **Ans. 7.5**

Complex Numbers

1. $(x + i)(x - i) = 10 \rightarrow x^2 - i^2 = 10 \rightarrow x^2 + 1 = 10$, $x^2 = 9$, $x = \pm 3$. **Ans. ± 3**

$$2. \left(\frac{5+2i}{2-i}\right)\left(\frac{3+4i}{3+4i}\right) - \left(\frac{1+i}{3+4i}\right)\left(\frac{2-i}{2-i}\right) = \frac{15+26i-8-(2-3i-1)}{6+5i+4} = \frac{6+29i}{10+5i} \cdot \frac{10-5i}{10-5i}$$

$$= \frac{60+290i-30i+145}{100+25} = \frac{205+260i}{125} = \frac{41}{25} + \frac{52}{25}i. \quad \text{Ans. } \frac{41}{25} + \frac{52}{25}i$$

3. One of the solutions may be real. $-3 \overline{) 3 \quad 11 \quad 11 \quad 15}$ Thus $3x^2 + 2x + 5 = 0$. Using
 $\quad \quad \quad -9 \quad -6 \quad -15$ the quadratic formula:

$$\begin{array}{r} 3 \quad 2 \quad 5 \quad 0 \\ x = \frac{-2 \pm \sqrt{4-4(15)}}{6} = \frac{-2 \pm \sqrt{-56}}{6} = \end{array}$$

$$\frac{-2 \pm 2\sqrt{14}i}{6} = -\frac{1}{3} \pm \frac{\sqrt{14}}{3}i. \quad \text{Ans. } -\frac{1}{3} \pm \frac{\sqrt{14}}{3}i$$

Arithmetic with percent

1. $10,000 (1.1)(1.1) = 12,100$. $12,100(.9)(.9) = 9801$. **Ans. 9801**

2. There are 10 palindromic numbers from 100 to 199. There are 10 palindromic numbers from 200 to 299, etc. This continues through the 900 numbers. $90/900 = 10\%$. **Ans. 10%**

3. Looking at perfect squares above 2000, 2025 is the only reasonable one. The square root of 2025 is 45. In the year 2000, the person would have been 20. 46^2 is 2116. **Ans. 20**

Team

1. There are 15 primes from 1 to 50. $15/50 = 3/10$. **Ans. 3/10**

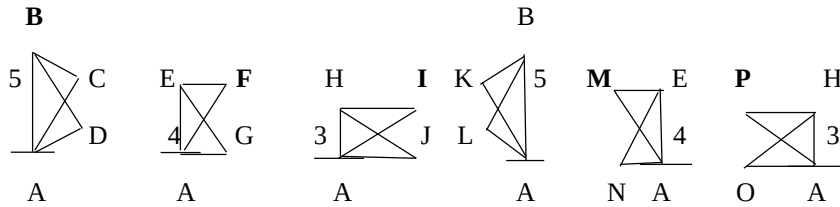
2. $(2+i)(3-i)(5+2i) = (6+i+1)(5+2i) = (7+i)(5+2i) = 35+19i-2 = 33+19i$.
 So $A = 33$ and $B = 19$. $A+B = 52$. **Ans. 52**

3. Starting with (1,1), (1,2), (1,3), (1,4), (1,5), (1,6). Of these 3 have a product that is odd. Of the next group using 2 as the first number all are even. So 3 of 12 are odd. Using 3, 4 and 5, 6 will produce the same result. $3/12 = 1/4 = 25\%$. **Ans. 25%**

4. There are 6^3 ordered outcomes. There are three arrangements of 1,1 and 4. There are 6 arrangements of 1, 2 and 3. There is only one version of 2, 2 and 2. Total = 10. $10/216 = 5/108$. **Ans. 5/108**

5. T_n has base vertices of $(4n, 0)$ and $(4n+4, 0)$. The midpoint of the base is $(8n+4, 0)/2 = 4n+2$. The altitude of the triangle is $y = 2x = 2(4n+2) = 8n+4$. The area of the isosceles triangle is $\frac{1}{2} \text{ base} \cdot \text{height} = \frac{1}{2}(4)(8n+4) = 16n+8$. **Ans. 16n + 8**

6. Graphing and establishing points, especially those beyond the circle with a radius of 4 from the origin: **B**



B, F, I, M and P are each 5 from the origin. A to P is 16 points. $5/16 = 31.25\%$. **Ans. 31.25%**

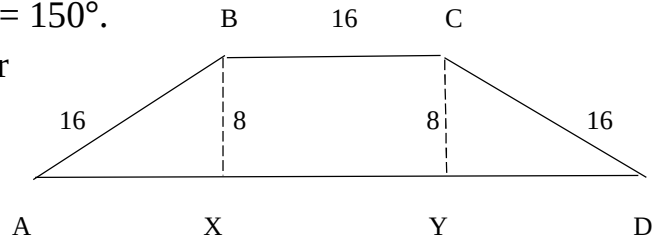
7. A to I is 12. $360^\circ/12 = 30^\circ$ degrees, an exterior angle. Each interior angle is 150° .

Quadrilateral ABCD is a trapezoid. $m\angle B = m\angle C = 150^\circ$.

$m\angle A = m\angle D = 30^\circ$. So if we drop a perpendicular from B to $\overline{AD}$ and C to $\overline{AD}$ to meet at X and Y respectively, then $XY = 16$, $AX = YD = 8\sqrt{3}$

and $BX = 8$. The area of ABCD =

$$\frac{1}{2}(8)(32 + 16\sqrt{3}) = 128 + 64\sqrt{3}.$$



Ans. $128 + 64\sqrt{3}$

8. $(x - 1 - 2i)^4 = 225 - 16(x - 1 - 2i)^2 \rightarrow$ Let $u = x - 1 - 2i$, then $u^4 = 225 - 16u^2 \rightarrow$
 $u^4 + 16u^2 - 225 = 0 \rightarrow (u^2 + 25)(u^2 - 9) = 0$. (1) $u^2 = -25$ or (2) $u^2 = 9$. In (1) $u = \pm 5i$,
 $x - 1 - 2i = \pm 5i \rightarrow x = 1 + 2i \pm 5i = 1 + 7i$ or $1 - 3i$. In (2) $u = \pm 3$, $x - 1 - 2i = \pm 3 \rightarrow$
 $x = 1 + 2i \pm 3 \rightarrow x = 4 + 2i$ or $-2 + 2i$.

Ans. $1 + 7i$ or $1 - 3i$ or $4 + 2i$ or $-2 + 2i$

9. $\sqrt{12 - 4\sqrt{5}} = \sqrt{A} - \sqrt{B}$, squaring: $12 - 4\sqrt{5} = A - 2\sqrt{AB} + B$. So (1) $A + B = 12$ and
 $\sqrt{AB} = 2\sqrt{5}$, so (2) $AB = 20$. In (1) $A = 12 - B$, thus in (2) $(12 - B)B = 20 \rightarrow 12B - B^2 = 20 \rightarrow$
 $B^2 - 12B + 20 = 0 \rightarrow (B - 10)(B - 2) = 0$, Thus $B = 10$ and $A = 2$, or $B = 2$ and $A = 10$. Since
 $\sqrt{12 - 4\sqrt{5}}$ must be positive, then A must be 10 and $B = 2$. $A - B = 8$. **Ans. 8**

Probability

1. $\frac{2}{3}$
2. 17
3. 117/1024

Exponents and Radicals

1. -2
2. 113
3. $\pm 1, \pm \frac{1}{32}$

Lines, Angles and Polygons

1. 540°
2. 490 cm (490)
3. 7.5 ($7\frac{1}{2}$)

Complex Numbers

1. ± 3
2. $\frac{41}{25} + \frac{52}{25}i$ ($\frac{41+52i}{25}$)
3. $-\frac{1}{3} \pm \frac{\sqrt{14}}{3}i$ ($\frac{-1 \pm \sqrt{14}i}{3}$)

Arithmetic with Percent

1. 9801 (9801 Pennies)
2. 10% (10)
3. 20

Team

1. $\frac{3}{10}$ (30%) (.3)
2. 52
3. 25% (.25) ($\frac{1}{4}$)
4. $\frac{5}{108}$
5. $16n + 8$
6. 31.25% (31.25) ($31\frac{1}{4}\%$) ($31\frac{1}{4}$)
7. $128 + 64\sqrt{3}$ (238.8513)
8. $1 + 7i$ or $1 - 3i$ or $4 + 2i$ or $-2 + 2i$
9. 8

Answers granted on appeal

- Round 1 #3: $\frac{27}{256}$ and $\frac{243}{4096}$ were both accepted, as the problem did not explicitly indicate a player could only win on their turn, as well as ambiguity about whether a player's turn was "until" they won.
- Team # 6: $16n - 8$ was also accepted as the problem had inconsistency in the labeling of the triangles.